Moriah College
בית ספר הר המוריה
CELEBRATING 80 YEARS 1943-2023 | 5703-5783

Student Number

2023 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time - 10 minutes
- Working time - 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number on the Question 11 Writing Booklet attached

Total marks: 69

Section I - 8 marks (pages 2-6)

- Attempt Questions 1-8
- Allow about 15 minutes for this section

Section II - 61 marks (pages 7-14)

- Attempt Questions 9-12
- Allow about 1 hour 45 minutes for this section

Section I

8 marks
Attempt Questions 1-8
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-8

---

1 Given $\overrightarrow{OA} = \underset{\sim}{i} + 2\underset{\sim}{j}$ and $\overrightarrow{AB} = 3\underset{\sim}{i} - \underset{\sim}{j}$, which is the correct value for $\overrightarrow{OB}$?

A. $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$

B. $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

C. $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

D. $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

2 What is the remainder when $P(x) = x^3 + 2x^2 - 5$ is divided by $x - 1$?

A. -2

B. -1

C. 0

D. 1

3 A sporting team needs 5 members, which must include at least two students from Year 11 and at least two students from Year 12.

There are ten Year 11 students and fifteen Year 12 students available for selection.

In how many distinct ways can the team be chosen?

A. 10

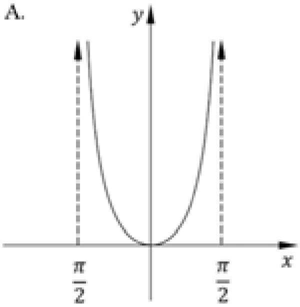
B. 5775

C. 33 075

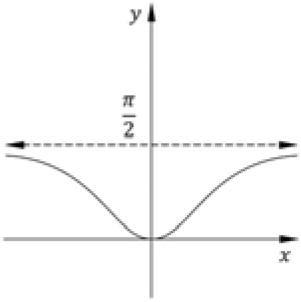
D. 396 900

4 Which of the following graphs best shows $y = \tan^{-1}(x^2)$?

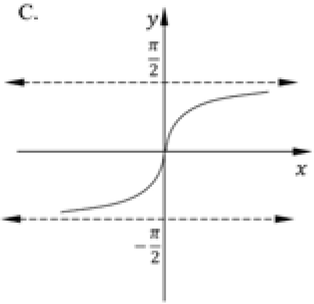
A.



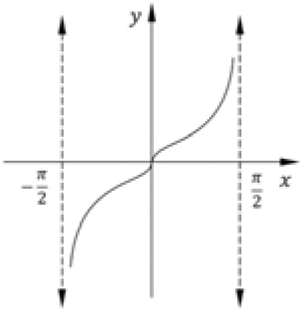
B.



C.



D.



5 A graph has parametric equations $x = \sin t$, $y = \cos^2 t + 1$.

What is its Cartesian equation?

A. $y = -x^2 + 2$ for $0 \leq x \leq 1$

B. $y = -x^2 + 2$ for $-1 \leq x \leq 1$

C. $y = x^2 - 2$ for $0 \leq x \leq 1$

D. $y = x^2 - 2$ for $-1 \leq x \leq 1$

6 A, B and C are collinear points with position vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ respectively. B lies between A and C. Given that $|\overrightarrow{BC}| = \frac{1}{2}|\overrightarrow{AB}|$, which of the following expressions is equal to $\underset{\sim}{c}$?

A. $\frac{1}{2}\underset{\sim}{a} - \frac{3}{2}\underset{\sim}{b}$

B. $\frac{3}{2}\underset{\sim}{a} - \frac{1}{2}\underset{\sim}{b}$

C. $\frac{3}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$

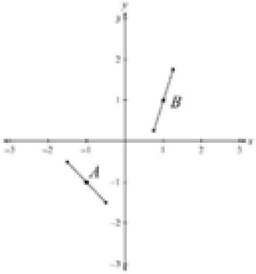
D. $\frac{3}{2}\underset{\sim}{b} - \frac{3}{2}\underset{\sim}{a}$

7 A direction field is to be drawn for the differential equation:

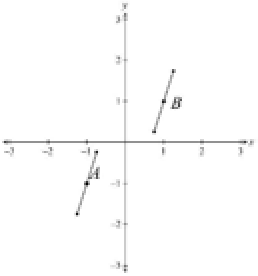
$$\frac{dy}{dx} = 2x + \frac{y}{x}.$$

Which of the graphs below shows the correct slope lines at the points A $(-1, -1)$ and B $(1, 1)$?

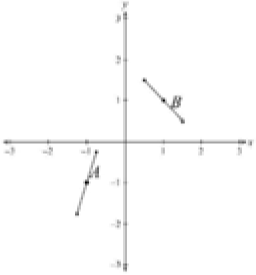
A.



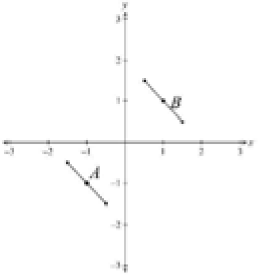
B.



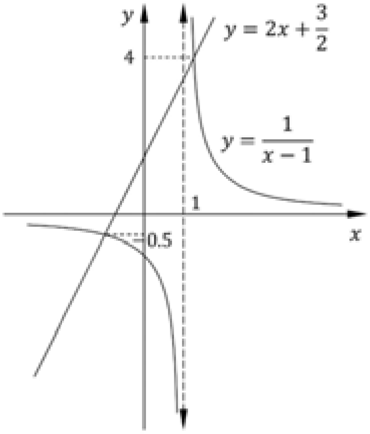
C.



D.



8 The graphs of $y = \frac{1}{x-1}$ and $y = 2x + \frac{3}{2}$ are shown below.



The functions intersect at y-values of -0.5 and 4.

Which of the following is the correct solution to $2x + \frac{3}{2} < \frac{1}{x-1}$?

A. $\left(-\infty, -1\right) \cup \left(1, \frac{5}{4}\right)$

B. $\left(-\infty, -\frac{1}{2}\right) \cup \left(0,4\right)$

C. $\left(-1,1\right) \cup \left(\frac{5}{4}, \infty\right)$

D. $\left(-\frac{1}{2}, 0\right) \cup \left(4, \infty\right)$

Section II

61 marks
Attempt Questions 9-12
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 9 (15 marks) Use the Question 9 Writing Booklet

(a) The quadratic equation $x^2 + 3x + 1 = 0$ has roots α and β.

(i) Find the value of $\alpha + \beta$. **1**

(ii) Find the value of $\alpha\beta$. **1**

(iii) Hence, or otherwise, find the value of $\alpha^2 + \beta^2$. **2**

(b) Evaluate **2**

$$\int_0^{\sqrt{2}} \frac{dx}{\sqrt{4 - x^2}}$$

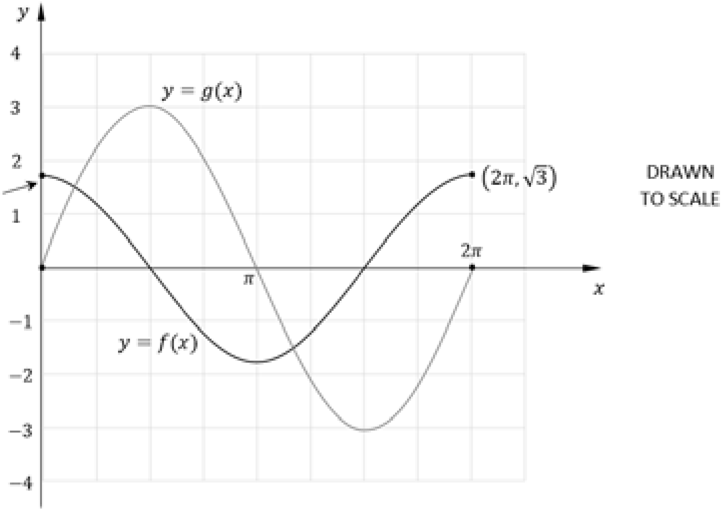
(c) Evaluate exactly $\cos^{-1}(\sin \frac{4\pi}{3})$. **2**

(d) (i) Write down the expansion of $(1 + x)^3$. **1**

(ii) Hence, find the coefficient of x^4 from $(2x - 1)^2(1 + x)^3$ **2**

Question 9 (continued)

(e) The graphs of the trigonometric curves $y = f(x)$ and $y = g(x)$ are shown below. **4**



Consider the curve given by $y = f(x) + g(x)$.

The equation of this curve can be expressed in the form $y = R\cos(x - \alpha)$
Find R and α.

End of Question 9

Question 10 (15 marks) Use the Question 10 Writing Booklet

(a) Use the substitution $u = x^2 + 1$, or otherwise, to find **2**

$$\int x\sqrt{x^2+1}\,dx.$$

(b) Lily and Ben are two of eight people.

(i) Find the number of ways that the 8 people can arrange themselves in a circle so that Lily and Ben are together. **1**

(ii) Find the number of ways that the 8 people can arrange themselves in a straight line so that Lily is somewhere to the left of Ben. **1**

(c) The vector $\underset{\sim}{u}$ starts at the origin and ends at $3\underset{\sim}{i} + 2\underset{\sim}{j}$.

The vector $\underset{\sim}{v}$ starts at $2\underset{\sim}{j}$ and ends at $3\underset{\sim}{i} + \lambda\underset{\sim}{j}$.

(i) Given that $\underset{\sim}{u}$ is perpendicular to $\underset{\sim}{v}$, show that $\lambda = \frac{-5}{2}$. **3**

(ii) Hence find the length of $\left|\underset{\sim}{v}\right|$. **2**

Question 10 (continued)

(d) A bowl of soup is heated to $85°C$. It is left to sit at room temperature.

The temperature of the soup after t minutes is given by

$$T = 25 + Ae^{kt}\ .$$

The soup reaches a temperature of $65°C$ after ten minutes.

(i) Find the value of A. **1**

(ii) Find the temperature of the soup after twenty minutes (answer to the nearest degree.) **2**

(e) Using the substitution $t = tan\frac{\theta}{2}$, solve the equation below, for $0 \leq \theta \leq 2\pi$ **3**

$$\sec\theta - 2\tan\theta = 1$$

Question 11 (14 marks) Use the Question 11 Writing Booklet

(a) Consider the identity

$$(2^1 + 2) + (2^2 + 4) + (2^3 + 6) + \ldots + (2^n + 2n) = 2^{n+1} + n(n+1) - 2.$$

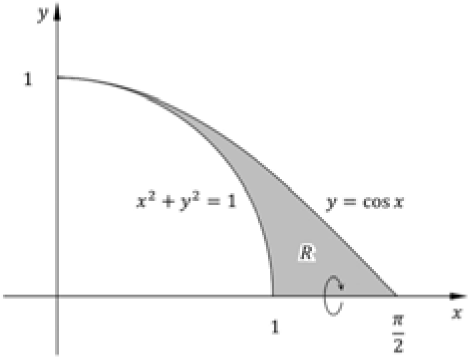
(i) Show that it is true for $n = 1$ **1**

(ii) Show that it is true for $n = 2$ **1**

(iii) By mathematical induction, show that it is true for $n \geq 1$ **2**

(b) The region R is bounded by the x-axis, the curve $y = \cos x$ and the unit circle, as shown in the diagram. **3**

The graph of $y = \cos x$ lies on or outside the unit circle.



Find the exact volume of the solid of revolution formed when the region R is rotated about the x-axis.

(c) The probability that Claire correctly chooses the winner of a game is 0.3.

(i) What is the probability that Claire will choose the winner of five out of eight games? **1**

(ii) What is the probability that Claire will choose the winner of at least two games out of the eight games. **2**

(d) Records show that 64% of students at a school travel to and from school by bus. A sample of 100 students at the school was taken to determine the proportion who travel to and from school by bus.

(i) Given that the mean of the sample proportion is 0.64, show that the standard deviation of the sample proportion is 0.048. **1**

(ii) Use the table below of $P(Z<z)$, where Z has a standard normal distribution, to estimate the probability that a sample of 100 students will contain at least 58 and at most 64 students who travel to and from school by bus. **3**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

End of Question 11

Question 12 (17 marks) Use the Question 12 Writing Booklet

(a) A box contains 3 blue marbles, 5 green marbles and *k* yellow marbles.
A marble is chosen at random and then placed back into the box. This is repeated four times.

(i) What is the probability that the first marble chosen is blue? **1**

(ii) Let *X* be the number of blue marbles chosen.

Find the smallest value of *k* for which $\text{Var}(X) < 0.8$ **3**

(b) The velocity, v, of a particle satisfies the following differential equation.

$$\frac{dx}{dt} = e^{-(2t+x)}.$$

(i) If the particle is initially at the origin, show that the displacement is given by **3**

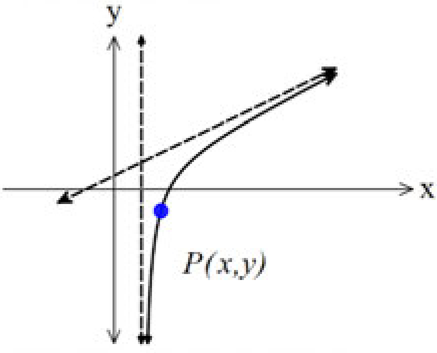
$$x = \ln\left[\frac{3 - e^{-2t}}{2}\right].$$

where, x, is in metres and the time, t, is in seconds.

(ii) Find the displacement of the particle when the velocity is $\frac{2}{23}$ m/s (answer to 2 decimal places). **3**

Question 12 (continued)

(c) The right hand section of the curve $y = \frac{x^2-4}{x-1}$ is drawn below.



The graph shows a vertical asymptote and another asymptote (oblique).

(i) Find the equation of the vertical asymptote. **1**

(ii) By dividing $x^2 - 4$ by $x - 1$, find the equation of the oblique asymptote **2**

(iii) The point $P(x, y)$ is shown on the graph.

This point moves along the curve so that its y-coordinate is changing at the rate of 2 units per second. (ie $\frac{dy}{dt} = 2$)

Find the rate at which the x-coordinate of P is changing at the instant when P meets the x-axis. **3**

(iv) Show that there are no points P on the section of curve where the rate of change of the x-coordinate equals the rate of change of the y-coordinate. **1**

End of Paper

2023 Trial HSC Mathematics Extension 1 Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	D
2	A
3	C
4	B
5	B
6	C
7	A
8	A

Worked Solutions

1 D

$$\begin{aligned}\overrightarrow{OB} &= \overrightarrow{OA} + \overrightarrow{AB} \\ &= \begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}3\\-1\end{pmatrix} \\ &= \begin{pmatrix}4\\1\end{pmatrix}, \qquad \text{so D.}\end{aligned}$$

2 A

$P(1) = (1)^3 + 2(1)^2 - 5 = -2,$ so A.

3 C

${}^{10}C_2 \times {}^{15}C_3 + {}^{10}C_3 \times {}^{15}C_2 = 33\,075,$ so C.

4 B

Since x^2 is even $y = \tan^{-1}(x^2)$ must be even, so eliminate C and D.

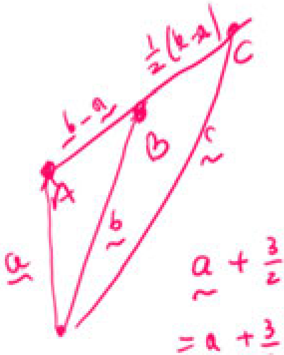
The range of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ the range of $y = \tan^{-1}(x^2)$ must lie within the same range, so eliminate A, leaving B.

5 B

$$x = \sin t \quad \rightarrow \quad -1 \le x \le 1$$

$$\begin{aligned} x^2 &= \sin^2 t \\ y - 1 &= \cos^2 t \\ \sin^2 t + \cos^2 t &= 1 \qquad \text{Pythagorean Identity} \\ x^2 + y - 1 &= 1 \\ y &= -x^2 + 2 \qquad \text{for } -1 \le x \le 1, \qquad \text{so B.}\end{aligned}$$

6 C



$$\underset{\sim}{a} + \frac{3}{2}(\underset{\sim}{b} - \underset{\sim}{a})$$

$$= \underset{\sim}{a} + \frac{3}{2}\underset{\sim}{b} - \frac{3}{2}\underset{\sim}{a}$$

$$= \frac{3}{2}\underset{\sim}{b} - \frac{\underset{\sim}{a}}{2}$$

7 A

at (1,1)

$$\frac{dy}{dx} = 2 + 1$$
$$= 3$$

at B ⤢ positive

at (−1, −1)

$$\frac{dy}{dx} = 2(-1) + 1$$
$$= -2 + 1$$
$$= -1$$

at C ⤡ negative

8 A

At the points of intersection:

$$2x + \frac{3}{2} = -\frac{1}{2} \rightarrow 2x = -2 \rightarrow x = -1$$

$$2x + \frac{3}{2} = 4\ 8 \rightarrow 2x = \frac{5}{2} \rightarrow x = \frac{5}{4}$$

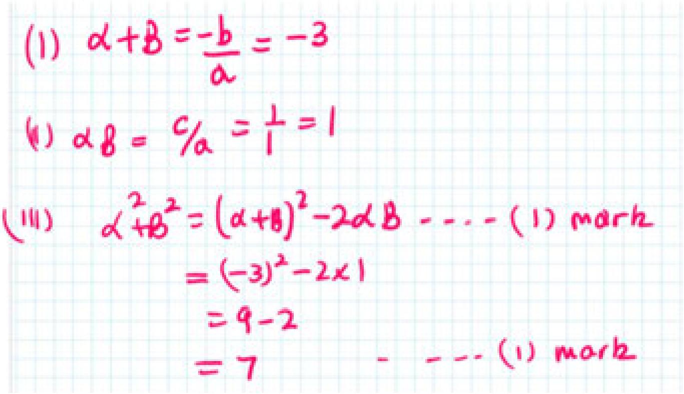
There is also the vertical asymptote at $x = 1$ to take into account. We need the x-values (not the y-values) for which the line is underneath the hyperbola.

$\therefore \left(-\infty, -1\right) \cup \left(1, \frac{5}{4}\right),$ so A.

Section II

Question 9

(a)



(1) $\alpha+\beta = \frac{-b}{a} = -3$

(ii) $\alpha\beta = c/a = \frac{1}{1} = 1$

(iii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ---- (1) mark

$= (-3)^2 - 2\times 1$

$= 9-2$

$= 7$ - --- (1) mark

(b)

$$\int_0^{\sqrt{2}} \frac{dx}{\sqrt{4-x^2}}$$

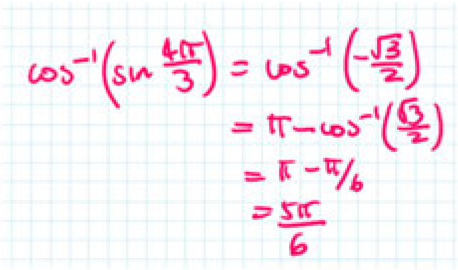
$$= \left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^{\sqrt{2}} \qquad \boxed{A}$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1}(0)$$

$$= \frac{\pi}{4} - 0$$

$$= \frac{\pi}{4} \qquad \boxed{B}$$

(c)



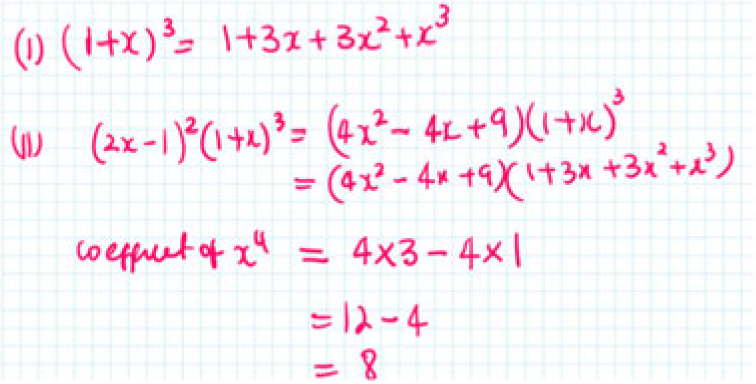
$\cos^{-1}\left(\sin\frac{4\pi}{3}\right) = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= \pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \pi - \pi/6$

$= \frac{5\pi}{6}$

(d)



(i) $(1+x)^3 = 1+3x+3x^2+x^3$

(ii) $(2x-1)^2(1+x)^3 = (4x^2-4x+9)(1+x)^3$

$= (4x^2-4x+9)(1+3x+3x^2+x^3)$

coeffient of x^4 $= 4\times3 - 4\times1$

$= 12-4$

$= 8$

(e)

$$f(x) = \sqrt{3}\cos x$$

$$g(x) = 3\sin x$$

$$f(x) + g(x) = \sqrt{3}\cos x + 3\sin x = R\cos(x-\alpha)$$

$$R = \sqrt{\left(\sqrt{3}\right)^2 + 3^2}$$

$$= \sqrt{12}$$

$$= 2\sqrt{3} \qquad \boxed{A} \text{ or } \boxed{B}$$

$$\tan(\alpha) = \frac{3}{\sqrt{3}}$$

$$\alpha = \tan^{-1}\left(\sqrt{3}\right)$$

$$= \frac{\pi}{3} \qquad \boxed{B} \text{ or } \boxed{A}$$

$$\therefore \ y = 2\sqrt{3}\cos\left(x - \frac{\pi}{3}\right)$$

Award full marks once they have found R and α correctly - no need to put in $R\cos(x-\alpha)$ form.

Question 10

(a)

$$\int x\sqrt{x^2+1}dx \qquad \boxed{\begin{aligned} u &= x^2+1 \\ \frac{du}{dx} &= 2x \\ dx &= \frac{du}{2x} \end{aligned}} \qquad \boxed{A}$$

$$= \int x\sqrt{u} \times \frac{du}{2x}$$

$$= \frac{1}{2}\int u^{\frac{1}{2}}\,du \qquad \boxed{B}$$

$$= \frac{1}{2} \times \frac{2}{3} \times u^{\frac{3}{2}} + c$$

$$= \frac{\sqrt{(x^2+1)^3}}{3} + c \qquad \boxed{C}$$

(b)

(i) $6! \times 2 = 1440$

(ii) $\frac{8!}{2!} = 20160$

(c)

$\underset{\sim}{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad \underset{\sim}{v} = \begin{pmatrix} 3 \\ \lambda \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ \lambda - 2 \end{pmatrix}$ (1) mark

(i) $\underset{\sim}{u} \cdot \underset{\sim}{v} = 0$

$3 \times 3 + 2(\lambda - 2) = 0$ 1 mark

$9 + 2\lambda - 4 = 0$

$2\lambda = -5$

$\lambda = -5/2$ 1 mark

(ii) $|\underset{\sim}{v}| = \left| \begin{matrix} 3 \\ -5/2 - 2 \end{matrix} \right|$

$= \left| \begin{matrix} 3 \\ -9/2 \end{matrix} \right|$ 1 mark

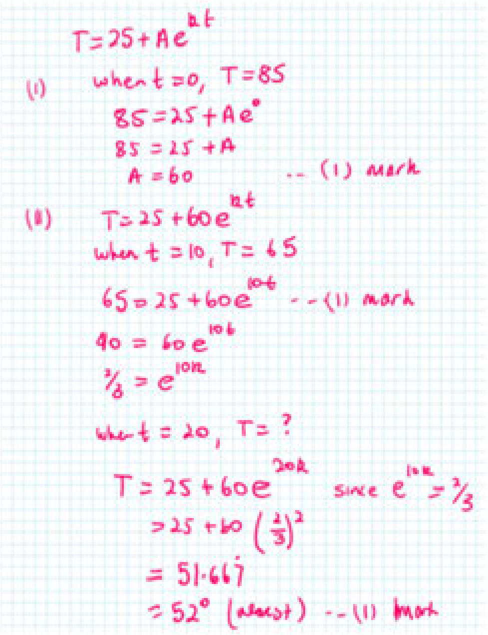
$= \sqrt{3^2 + (-9/2)^2}$

$= \sqrt{\frac{117}{4}}$

$= \frac{1}{2}\sqrt{117}$

$= \frac{3\sqrt{13}}{2}$ 1 mark

(d)



$T = 25 + Ae^{kt}$

(i) when $t = 0$, $T = 85$

$85 = 25 + Ae^{0}$

$85 = 25 + A$

$A = 60$ -- (1) mark

(ii) $T = 25 + 60e^{kt}$

when $t = 10$, $T = 65$

$65 = 25 + 60e^{10k}$ -- (1) mark

$40 = 60e^{10k}$

$2/3 = e^{10k}$

when $t = 20$, $T = ?$

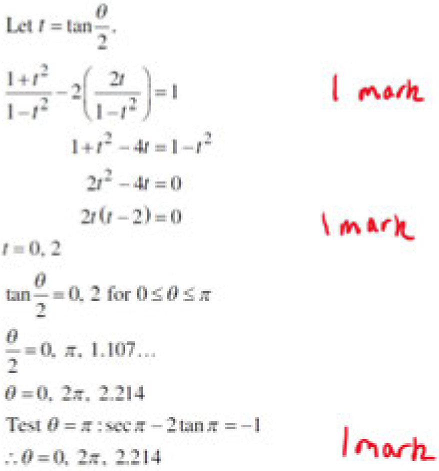
$T = 25 + 60e^{20k}$ since $e^{10k} = 2/3$

$= 25 + 60\left(\frac{2}{3}\right)^2$

$= 51.66\dot{7}$

$= 52°$ (nearest) -- (1) mark

(e)



Let $t = \tan\frac{\theta}{2}$.

$\frac{1+t^2}{1-t^2} - 2\left(\frac{2t}{1-t^2}\right) = 1$ 1 mark

$1 + t^2 - 4t = 1 - t^2$

$2t^2 - 4t = 0$

$2t(t-2) = 0$ 1 mark

$t = 0, 2$

$\tan\frac{\theta}{2} = 0, 2$ for $0 \le \theta \le \pi$

$\frac{\theta}{2} = 0, \pi, 1.107\ldots$

$\theta = 0, 2\pi, 2.214$

Test $\theta = \pi : \sec\pi - 2\tan\pi = -1$

$\therefore \theta = 0, 2\pi, 2.214$ 1mark

Question 11

(a) (i)

$LHS = 2^1+2$

$= 4$

$n=1$

$RHS = 2^{1+1}+1(1+1)-2$

$= 4+2-2$

$= 4$

$\therefore LHS = RHS$

1mark

(ii)

$LHS = (2^1+2)+(2^2+4)$

$= 4+4+4$

$= 12$

$n=2$

$RHS = 2^{2+1}+2(2+1)-2$

$= 8+6-2$

$= 12$

$\therefore$ 1mark

$\therefore LHS = RHS$

(iii)

Let $P(n)$ represent the proposition.

If $P(k)$ is true for some arbitrary $k \geq 1$, then

$$(2^1+2)+(2^2+4)+(2^3+6)+...+(2^k+2k) = 2^{k+1}+k(k+1)-2$$

RTP $P(k+1)$:

$$(2^1+2)+(2^2+4)+(2^3+6)+...+(2^{k+1}+2(k+1)) = 2^{k+2}+(k+1)(k+2)-2$$

$$\begin{aligned} \text{LHS} &= 2^{k+1}+k(k+1)-2+2^{k+1}+2(k+1) \text{ from } P(k) && \boxed{B} \\ &= 2(2^{k+1})+(k+1)(k+2)-2 \\ &= 2^{k+2}+(k+1)(k+2)-2 && \boxed{C} \\ &= \text{RHS} \end{aligned}$$

$\therefore P(k) \Rightarrow P(k+1)$

Hence $P(n)$ is true for $n \geq 1$ by induction.

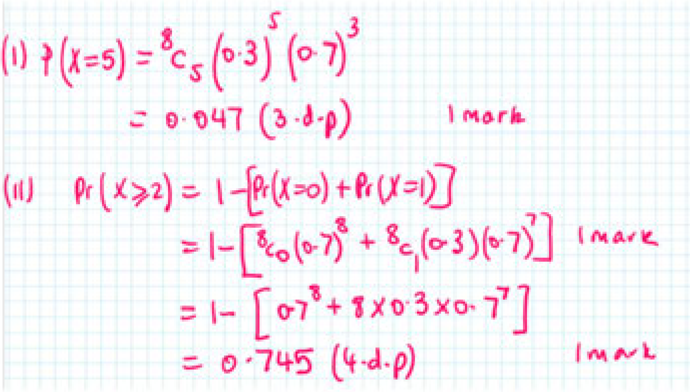
(b)

The unit circle intercepts the x-axis at $(1,0)$ and $y = \cos x$ intercepts the x-axis at $\left(\frac{\pi}{2}, 0\right)$.

Rotate the area under $y = \cos x$ about the x-axis then subtract

$$\begin{aligned} V &= \pi\int_0^{\frac{\pi}{2}} \cos^2 x \, dx - \frac{2}{3}\pi(1)^3 && \boxed{A} \\ &= \frac{\pi}{2}\int_0^{\frac{\pi}{2}} (\cos 2x+1) \, dx - \frac{2\pi}{3} \\ &= \frac{\pi}{2}\left[\frac{\sin 2x}{2}+x\right]_0^{\frac{\pi}{2}} - \frac{2\pi}{3} && \boxed{B} \\ &= \frac{\pi}{2}\left(\left(0+\frac{\pi}{2}\right)-(0+0)\right)-\frac{2\pi}{3} \\ &= \frac{\pi^2}{4}-\frac{2\pi}{3} \text{ units}^3 && \boxed{C} \end{aligned}$$

(c)



(i) $P(X=5) = {}^8C_5(0\cdot3)^5(0\cdot7)^3$

$= 0\cdot047$ (3·d·p) 1 mark

(ii) $Pr(X\geqslant2) = 1-[Pr(X=0)+Pr(X=1)]$

$= 1-[{}^8C_0(0\cdot7)^8 + {}^8C_1(0\cdot3)(0\cdot7)^7]$ 1 mark

$= 1-[0\cdot7^8 + 8\times0\cdot3\times0\cdot7^7]$

$= 0\cdot745$ (4·d·p) 1 mark

(d) (i)

For students in the sample, the number of students traveling to and from school by bus is a random variable X with the binomial distribution B (100, 0.64). Hence standard deviation $\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.64\times(1-0.64)}{100}} = 0.048.$ - - - 1 mark

(ii)

$$P(0.58 \leq \frac{X}{100} \leq 0.64)$$

Z scores for 0.64 $= \frac{0.64-0.64}{0.048} = 0$ - - - 1 mark

Z scores for 0.58 $= \frac{0.58-0.64}{0.048} = -1.25$ - - - - 1 mark

$$P(-1.25 \leq Z \leq 0) = P(Z \leq 1.25) - 0.5$$
$$= 0.8944 - 0.5$$
$$= 0.3944$$ - - - 1 mark

Question 12

(a)(i) $p = \frac{3}{8+k}$ --- 1 mark

(ii) $$1-p = 1-\frac{3}{8+k}$$

$$= \frac{5+k}{8+k}$$ - -

$$\mathrm{Var}(X) = np(1-p)$$

$$= 4\left(\frac{3}{8+k}\right)\left(\frac{5+k}{8+k}\right)$$ --- 1 mark

$$4\left(\frac{3}{8+k}\right)\left(\frac{5+k}{8+k}\right) < 0.8$$

$$\frac{12(5+k)}{(8+k)^2} < 0.8$$

$$60+12k < 0.8(8+k)^2$$ -- 1 mark

$$75+15k < 64+16k+k^2$$

$$k^2+k-11>0$$

$$k<-3.85,\ k>2.85$$

$$\therefore k=3$$ --- 1 mark

(b) (i)

$$\frac{dy}{dx} = e^{-(2x+y)}$$

$$e^y\,dy = e^{-2x}\,dx \qquad \boxed{A}$$

$$\int_0^y e^y\,dy = \int_0^x e^{-2x}\,dx$$

$$\left[e^y\right]_0^y = -\frac{1}{2}\left[e^{-2x}\right]_0^x \qquad \boxed{B}$$

$$e^y - 1 = -\frac{1}{2}\left(e^{-2x}-1\right) \qquad \boxed{C}$$

$$e^y = -\frac{e^{-2x}}{2}+\frac{1}{2}+1$$

$$= \frac{3-e^{-2x}}{2} \qquad \boxed{D}$$

$$y = \ln\left[\frac{3-e^{-2x}}{2}\right]$$

(ii)

$x = \ln\left[\frac{3-e^{-2t}}{2}\right]$

$$\left.\begin{aligned} v &= \frac{\frac{2e^{-2t}}{2}}{\frac{3-e^{-2t}}{2}} = \frac{2e^{-2t}}{3-e^{-2t}} \\ v &= \frac{2}{3e^{2t}-1} \end{aligned}\right\} \text{1mark}$$

$3e^{2t}-1 = \frac{2}{v}$

$3e^{2t} = \frac{2}{v}+1$

when $v = \frac{2}{23}$

$3e^{2t} = 24$

$e^{2t} = 8$ 1mark

$v = e^{-(2t+x)}$

$v = e^{-2t} \times e^{-x}$

$\frac{2}{23} = e^{-2t} \times e^{-x}$

$e^{-x} = \frac{2e^{2t}}{23}$

$x = -\ln\left(\frac{2e^{2t}}{23}\right)$

$x = -\ln\left(\frac{2\times 8}{23}\right)$

$x = -\ln\left(\frac{16}{23}\right)$

$x = \ln\left(\frac{23}{16}\right)$

$x = 0{\cdot}36\text{ m}$ .. 1mark

(c)

(i) $x = 1$ is vertical asymptote

(ii)
$$\begin{array}{r} x+1 \\ x-1\;\overline{)\,x^2-4} \\ \underline{x^2-x} \\ x-4 \\ \underline{x-1} \\ -3 \end{array}$$

1mark

$\therefore\ y = x+1 - \frac{3}{x-1}$

$y = x+1$ is oblique asymptote 1 mark

(iii) $\frac{dy}{dt} = 2$

$\frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$

$\frac{dx}{dt} = \frac{(x-1)^2}{(x-1)^2+3} \times 2$

$y = x+1 - \frac{3}{x-1}$

$$\left.\begin{aligned} \frac{dy}{dx} &= 1+3(x-1)^{-2} \\ &= 1+\frac{3}{(x-1)^2} \\ &= \frac{(x-1)^2+3}{(x-1)^2} \\ &= \frac{x^2-2x+1+3}{(x-1)^2} \\ &= \frac{x^2-2x+4}{(x-1)^2} \end{aligned}\right\} \text{1mark}$$

Now when P meets x-axis

$y = \frac{x^2-4}{x-1}$

when $y = 0$ $x = 2$ ... 1mark

$\therefore \frac{dx}{dt}(2) = \frac{(2-1)^2}{(2-1)^2+3} \times 2$

$= \frac{1}{4} \times 2$

$= 0{\cdot}5$ 1mark

(iv) $\frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dx}$

For $\frac{dx}{dt} = \frac{dy}{dt}$ $\therefore$ $\frac{dx}{dy}$ must equal to 1

however $\frac{dx}{dy} = 1 + \frac{3}{(x-1)^2}$

which never equals to 1

$\therefore \frac{dx}{dt} \neq \frac{dy}{dt}$